

California Solar Initiative

**RD&D** ■ Research, Development, Demonstration  
■ and Deployment Program



Final Project Report:

# Comprehensive System Assessment of Smart Grid-tied Energy Storage System Using Second-life Lithium Batteries



Grantee:

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# Preface

The goal of the California Solar Initiative (CSI) Research, Development, Demonstration, and Deployment (RD&D) Program is to foster a sustainable and self-supporting customer-sited solar market. To achieve this, the California Legislature authorized the California Public Utilities Commission (CPUC) to allocate **\$50 million** of the CSI budget to an RD&D program. Strategically, the RD&D program seeks to leverage cost-sharing funds from other state, federal and private research entities, and targets activities across these four stages:

- Grid integration, storage, and metering: 50-65%
- Production technologies: 10-25%
- Business development and deployment: 10-20%
- Integration of energy efficiency, demand response, and storage with photovoltaics (PV)

There are seven key principles that guide the CSI RD&D Program:

1. **Improve the economics of solar technologies** by reducing technology costs and increasing system performance;
2. **Focus on issues that directly benefit California**, and that may not be funded by others;
3. **Fill knowledge gaps** to enable successful, wide-scale deployment of solar distributed generation technologies;
4. **Overcome significant barriers** to technology adoption;
5. **Take advantage of California's wealth of data** from past, current, and future installations to fulfill the above;
6. **Provide bridge funding** to help promising solar technologies transition from a pre-commercial state to full commercial viability; and
7. **Support efforts to address the integration of distributed solar power into the grid** in order to maximize its value to California ratepayers.

For more information about the CSI RD&D Program, please visit the program web site at [www.calsolarresearch.ca.gov](http://www.calsolarresearch.ca.gov).

## **Acknowledgements**

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## Abstract

This report summarizes a demonstration of second life lithium ion batteries as a smart grid-tied photovoltaic battery energy system. A single family household demonstrated solar storage and demand side management, integrating a PV array, a grid interface and battery energy storage. The stationary battery pack, composed of retired vehicle traction batteries, served as an energy buffer accumulated excess PV generated energy during off-peak hours and discharged during peak hours. A battery management system was developed to mitigate battery imbalance issues via extended Kalman filter SoC estimator, enhanced high current shunting, and protective circuitry. The system provided a proof of concept that retired EV batteries can be reused for secondary applications, benefiting both transportation and building energy sectors. Three decision-table-based control strategies were demonstrated to test system functionality with different objectives. Test data obtained from the system located in Davis, CA indicates a 64% to 100% reduction in grid draw depending on weather and operational modes of the system. The system successfully achieved proposed goals demonstrating the utilization of used vehicle traction battery for second round of application, optimization of solar energy harvest and supporting electric vehicle charging.

## Executive Summary

### **Introduction**

Residential solar photovoltaic systems have grown in popularity over the past decade, due in part to government incentives, decreases in production costs, and, in many states, net energy metering (NEM) tariffs. NEM tariffs allow the customer-generator to treat the electricity grid as a storage device by exporting electricity when the onsite demand is less than the solar production and importing electricity from the grid when demand exceeds solar production. The benefits of NEM tariffs, in combination with other incentives and policy support mechanisms, have been a key factor in growing the market for residential photovoltaic systems in California (1). However, these programs, and similar programs, force electric utilities to accept all exported electricity and supply electricity in the absence of the solar generation, regardless of need or available resources. The addition of a PV array to a home significantly changes the demand profile of the home. While this change provides a meaningful decrease in the demand it can change the residence into a supplier of energy at different times of the day. As the penetration of distributed PV systems increases, utilities will need to begin following the changes in distributed PV generation and demand. Failure to consider the fluctuations in solar irradiance and the response of the PV arrays to voltage changes or other disturbances that could cause the PV array to shut down for self-preservation can have significant negative impacts on the distribution grid.

This work, was to develop and demonstrate control strategies to optimize the energy flow of the electrical energy storage using second life lithium ion batteries. This system can significantly reduce peak electricity use and can enable demand response (DR) if used in conjunction with a local grid service provider. In an extreme condition, the system can work as an uninterruptible power supply (UPS) providing the house with full electricity for up to a day. It also can efficiently manage the energy produced from PV panel instead of selling it back to the grid. The application of 2nd life battery is a timely topic with great market potential. However, few demonstration projects have been implemented, and limited performance data is available. This project presents the development and the demonstration of a PV battery integrated energy system performing solar storage and demand side management in a single family home.

### **Project goals**

The main goal of this project was to develop and demonstrate optimal control strategies for the smart grid-tied energy storage system. Developed control schemes have been experimentally evaluated in a residential home that is equipped with PV panels, smart appliances and a level 2 charging system for a plug-in hybrid electric vehicle. Cost and lifetime assessment of battery pack

also have been performed to examine the feasibility of using second life traction battery as grid tied energy storage. This project has proved the concepts of optimized solar energy use in residential home, transportation, communication between vehicles and the electric grid, and second-life applications for traction batteries. Relevant tasks included (1) Optimize Energy management Strategies for Different Objectives Including Demand Response, (2) Demonstrate the System with Modes 1, 2 and 3 for Long-Term Periods, (3) Assessment of the System Cost, Benefit and Environment Impact, and (4) Assessment of the Second Life Battery Pack State of Health Degradation.

### **Project task description**

The work described in this report was to develop and demonstrate a control strategy for the smart grid-tied energy storage system with the following characteristics:

- Powered primarily by solar PV panels and energy stored in second-life lithium ion traction batteries
- Two way AC/DC grid interface allowing for various energy flow between battery and grid electricity
- Reasonable cost and footprint for residential application

A control strategy was developed to optimize the electrical energy storage (EES) system operation. Two sets of management strategies (modes) have been first examined to minimize peak demand as well as to lower electricity cost. A decision table was constructed to control the power flow between each of the system components. Generally, in both modes, when the utility price is off-peak, the house load tends to be grid powered. When the utility price is at peak, the house loads tend to be PV or battery powered to avoid grid peak. When PV is over producing the controller will try to charge the battery or back feed to the grid if the grid price is at its peak. Three different control modes were developed in this work. The objective of the energy management Mode 1 is to optimize the amount of solar energy back feeding to the grid during the peak pricing hours while the Mode 2 is to support the house demand with minimum grid support possible. The objective of the energy management Mode 3 is to optimize the utilization of solar energy to support the house energy demand. Three decision making table based control algorithms were developed, targeted to improve 'peak shaving', 'grid dependency', and 'solar penetration' respectively. Each mode has been demonstrated for a minimum of a few weeks.

### **Project outcome**

This demonstration showed that a system with 10kWh battery pack and 2.16 kW PV array provided a versatile energy solution for a single family house. Varying by the usage demands and operation strategies, the system was able to achieve 64% to 100% reduction of grid usage, and



further improvement of solar penetration, with the help of battery storage matching the source and the demand. Over all, the Mode 3 aggressively stored solar energy to neutralize the grid interaction instead of directly sending back to the grid. This may be a relatively favorable energy management approach for the high solar penetration future grid. The system will continue to serve as a research platform, where advanced demand side management control strategies can be tested.

## **Conclusion**

This project presents the development and the demonstration of a PV battery integrated energy system performing solar storage and demand side management in a single family home. Three decision-table-based control strategies were demonstrated to test system functionality with different objectives. With proper design and engineering, used lithium batteries can be re-utilized as battery assemblies of competitive performance with the exception of imbalance at high state of charge, and lower round trip efficiency compared to systems with new batteries. A complete design document was presented including battery pack design and system integration. A battery management system was developed specifically for the 2nd life battery pack utilizing enhanced balancing circuitries to improve the battery pack performance. The system provided a proof of concept that retired EV batteries can be reused for secondary applications, benefiting both transportation and building energy sectors.

## **Recommendations**

This project demonstrated that 2nd life batteries can be repurposed as stationary storage for solar energy and demand side management. The commercial viability of this application is dependent upon the battery price, the grid storage market and the system long term performance. Longer time-period battery life cycle assessment testing will be conducted as an extension to this study. The size of the interconnected system for this research project was very small. With only one single family home and small scale roof top PV, the energy source and demand were constantly fluctuating due to various impacts, making exploring the potential for energy storage very challenging. Based on this study, three recommendations can be made: 1) 2nd life batteries may be repurposed as stationary energy storage given enhanced battery management to resolve issues in performance imbalance; 2) using battery module as the smallest building block, and deploying, cycling, and retiring the module independently help reduce the system cost while maintaining the overall system performance; 3) larger size (community to micro-grid scale) distributed energy source and demand are preferred to optimize the performance of a 2nd life battery pack.

## 1. INTRODUCTION

Second life batteries are batteries retired from their first application in plug-in hybrid electric vehicles (PHEVs) or electric vehicles (EVs) and repurposed for a second, typically lower performance application. The reduced performance application is generally required due to the imminent degradation that happens to batteries during their first application. According to the US Advanced Battery Consortium (USABC) standard for EV batteries, a battery cell has reached its end of life when the cell capacity has dropped below 80% of the rated capacity or the power density becomes less than 80% of the rated power density at 80% depth of discharge (DoD)[1]. For PHEVs, the impact of battery pack performance degradation is less significant, since the performance degradation of the battery pack due to aging can be compensated by the internal combustion engine (ICE). As a result, a PHEV battery may degrade more than the USABC standard specifies while still being able to provide value in an automotive application. Consequently, it is expected that battery cells with 80%, or less, of the rated capacity will be retired from PHEV/EV applications and will be available in the second life market. As PHEVs and EVs gain popularity the number of aged vehicle batteries will increase, posing recycling issues and making second life applications more attractive. Second use of lithium-ion traction battery applications is an applicable approach to extend the useful battery life. This aids in conserving resources and reducing environmental impacts, and is expected to have significant market potential as lithium-ion battery packs are beginning mass production for transportation use[2,3]. A second life battery pack, when properly sized, is able to deliver equivalent performance as a new battery pack, but at a larger volume and lower cost. Another important feature of a second life battery pack is, when cells of varying quantities of degradation are assembled together, the performance of the whole pack is governed by the weakest bank. The increased likelihood of battery bank capacity imbalance in second life battery packs increases the risk of over voltage and/or over current within the pack, and therefore requires a well-integrated battery management system[4-9]. As energy generation shifts from fossil fuels to alternative sources, energy storage will become an important component for grid stability and peak shifting, due to the improperly matched peak production of renewables versus grid demand [10-17]. Over the years, lithium ion battery applications have expanded from mobile electronics to automotive and aerospace. Popular candidates for battery stationary energy storage includes lithium batteries, lead acid batteries, flowing electrolyte batteries or sodium-beta high temperature batteries.

In this project, the second life battery pack was operated as an energy buffer shifting energy from times of peak PV production to times of peak energy consumption. The battery charge versus

discharge decision was made based on three system variables: 1) Time varying utility price, 2) Energy demand versus the PV production, and 3) Battery status. An example daily usage cycle typically has PV production occurring from 9am to 6pm, and any excess production will be stored in the battery pack. The typical energy usage peak occurs from 5pm to 9pm and typical utility time varying price peaks from 2pm to 8pm. During peak usage and peak utility price time periods, the battery tends to discharge to support the energy deficit. This energy flow management approach is a simple strategy for utilizing battery storage. When utility price is off peak, the energy demand will always be covered by the grid instead of battery. In addition, a one-day forecast is also implemented in the algorithm. This report summarizes findings from the demonstration of the three energy management schemes, tasks include 1) Optimizing energy management strategies for different objectives including demand response, 2) Demonstrating the system with modes 1, 2, and 3 for long-term periods, 3) Assessment of the system cost, benefit and environment impact, and 4) Assessment of the second life battery pack state of health degradation.

## **2. PROJECT OBJECTIVES**

The main goal of this project is to develop and demonstrate optimal control strategies for the smart grid-tied energy storage system. Developed control schemes have been experimentally evaluated in a residential home that is equipped with PV panels, smart appliances and a level 2 charging system for a plug-in hybrid electric vehicle. Cost and lifetime assessment of battery pack also have been performed to examine the feasibility of using second life traction battery as grid tied energy storage. This project has proved the concepts of optimized solar energy use in residential home, transportation, communication between vehicles and the electric grid, and second-life applications for traction batteries. Relevant tasks are presented below.

Task1-1: Optimize Energy Management Strategies for Different Objectives Including Demand Response

Task1-2: Demonstrate the System with Modes 1, 2 and 3 for Long-Term Periods

Task2-1: Assessment of the System Cost, Benefit and Environmental Impact

Task2-2: Assessment of the Second Life Battery Pack State of Health Degradation

### 3. PROJECT APPROACH

#### PV-EES Integrated home system

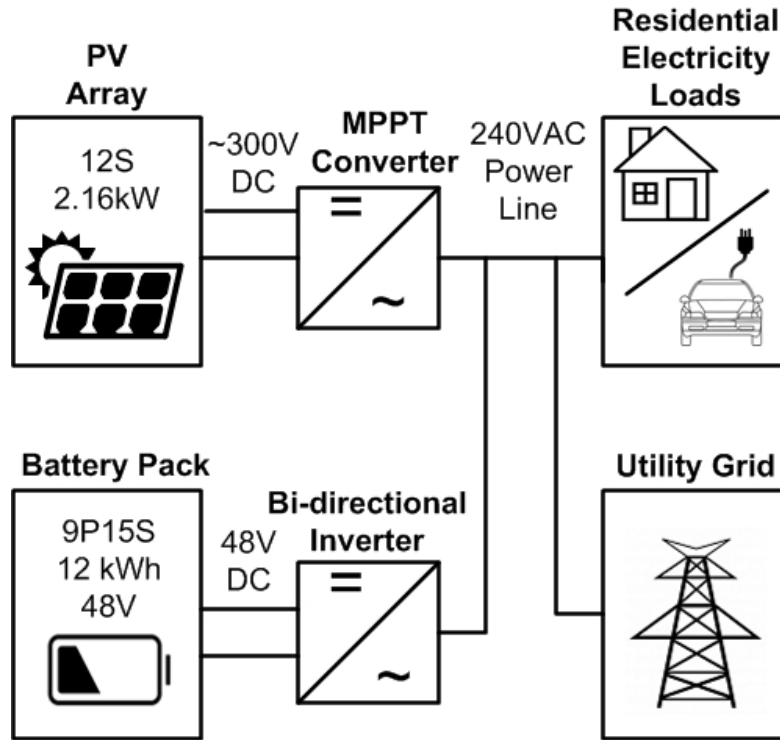
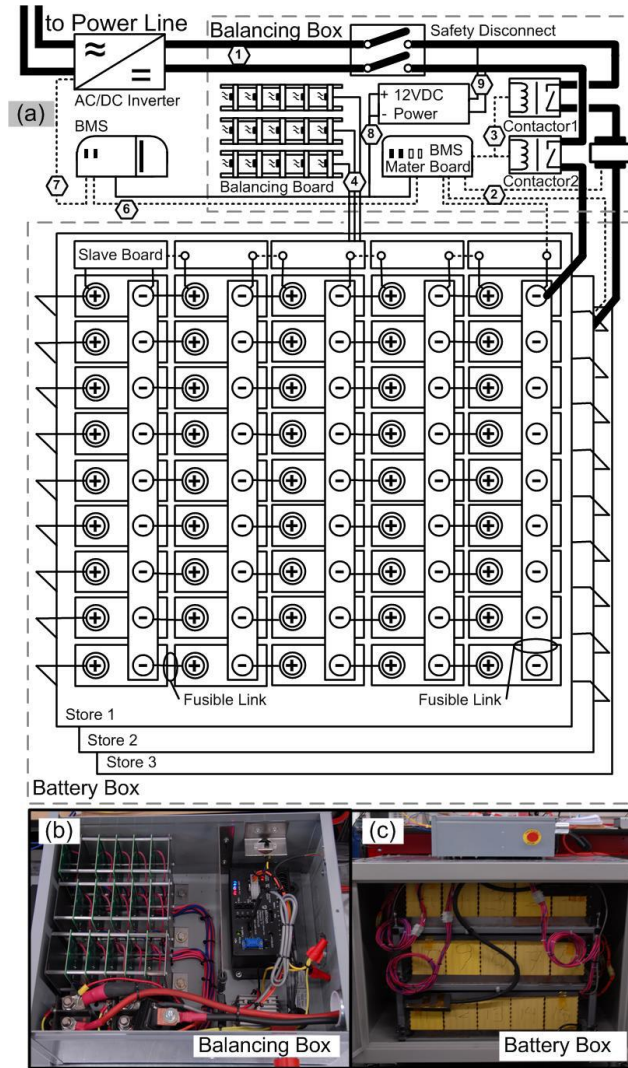


Figure 1 Diagram of the System Energy Management Layout.

As the system diagram illustrates in Fig. 1, a PV array with 12 panels in series was connected to the AC power line via two-step maximum power point tracing (MPPT) converter, providing 2.16 kW of rated power output. A battery pack was assembled using 135 units of 2nd life batteries, configured as nine batteries in parallel as one bank, and 15 banks in series. The battery pack provides 48V DC output, 5 kW maximum power capabilities and 12kWh total capacity. A 48 VDC-120 VAC bidirectional power converter is deployed to charge and discharge the battery pack. The PV and battery together formed an interconnected energy system, providing sustainable electricity source and storage for a single family home. The re-purposed prismatic batteries have the nominal capacity of 40Ah. After the application as vehicle traction batteries, these batteries have a remaining capacity of 20 to 30 Ah. Due to the nature of 2nd life batteries, the battery pack assembly possesses imbalance in state of charge (SoC), capacity, and internal resistance, all of which impair its electro performances. An intricate battery management system is applied including automated and manual disconnect, battery balancing and model based SoC and state of health (SoH) estimation.



**Figure 2 Battery pack (a) schematic diagram; (b) picture of the battery management system (on top of the battery pack); (c) picture of battery pack.**

The mechanical and electrical design aspects of the battery pack are summarized in Fig. 2. The batteries were interconnected via copper bus bars and 10AWG fusible links. The DC power line was protected by a DC safety disconnect (line 1) and two relays (line 3). A distributed master-slave battery management system (BMS) module was applied to measure voltage and temperature of battery cells, and current of battery pack (line 2). A boosted battery passive balancing circuitry was installed to provide 2A maximum current shunting for each battery bank (line 4). A micro controller reads battery measurements, and estimates the values of battery SoC, capacity and internal resistance for each bank (line 6). The BMS sends requests to the dc/ac inverter (line 7) to control the battery current input and output (line 1). A dc/dc converter powers the BMS (line 8) by sinking a small portion of energy from the battery (line 9). The assembled

battery pack was operated under the constraints of cell voltage range 2.8 to 3.65V, temperature limits 0 to 60 °C, and a current limit of 100A. Via design and engineering steps, this system managed to retrofit used EV batteries for a second round of application as stationary energy storage.

## Data Acquisition Service

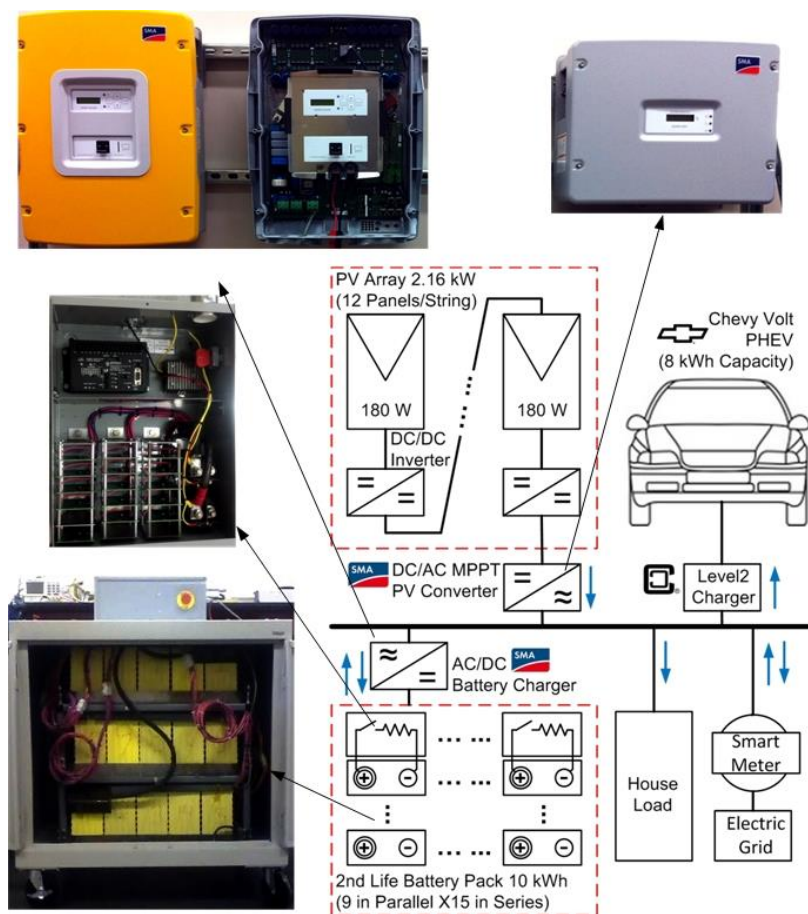


Figure 3 Diagram of system information network.

An intelligent information network was installed for data collection and analysis. As illustrated in Figure 3, a WirelessGlue™ gateway serves as the central gateway that receives information from the BMS, SMA®Webbox, Tigo® gateway, and ZigBee radios. The SMA®Webbox logs data of the SMA products, including the DC input from the battery pack, the AC output from the battery charger/discharger, and the AC output from the SMA MPPT PV converter. It also hosts a local HTTP server that can be continuously accessed through the central gateway (Line 4 in Figure 2). Similarly, the Tigo® gateway logs output data of each PV panel and transfers the data via wireless

communication to the central gateway (Line 7). ZigBee radios connected to the central gateway via Ethernet were installed in the house. They receive data from ZigBee equipped appliances such as smart plugs, smart meter, and a ClipperCreek® vehicle charger (Line 5 and Line 6). The BMS receives voltage, current, and temperature measurements of each battery bank through Line 1, and estimates battery SoC and SoH of the battery pack. Also, the BMS obtains the system operating data from the central gateway, including instant utility price, PV output, and house power demand. Based on the information, the BMS algorithm implements the design control decision, which is submitted to the central gateway (Line 2) and routed to the battery charger/discharger (Line 4) to operate the battery pack. Finally, the central gateway assembles all the data from different sources and sends the packaged system information to a server in the cloud.

Table 1 lists the servers the research team developed for data logging. The main Aggie Village Home Server is run at the WirelessGlue gateway providing a host for the BMS and stores data on the local database. The Tigo and SMA sever logs PV energy data. The Obvius smart panel server logs grid interaction data. The battery data server logs battery operation data.



Figure 4 Screen shot of web based data server: top, SME Webbox; middle, Obvius smart panel; bottom, TiGo system



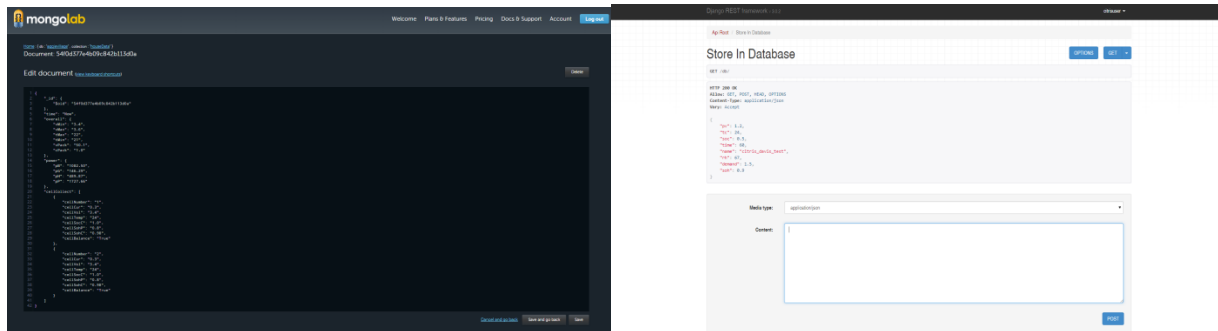
Figure 4 shows a screen shot of the different data logging servers. In addition, WirelessGlue provided the ZigBee data logger service to monitor the smart plug and ZigBee equipped appliance such as the vehicle charger. As a result, energy consumption of major appliances in the house can be individually monitored. This integrated information network allows researchers to keep good track of energy flow in the system. More importantly, it provides a convenient access for the users to check, manage and conserve their energy usage.

**Table 1. List of data logging server**

	Data Acquisition Service	Access
1)	Aggie Village Home Server via SSH protocol	<a href="sshgsf@ucdavisvillage.ddns.net">sshgsf@ucdavisvillage.ddns.net</a>
2)	Tigo Energy via Tigo live view service	<a href="http://www.tigoenergy.com/">http://www.tigoenergy.com/</a>
3)	SMA webbox server	<a href="http://ucdavisvillage.ddns.net:3334/">http://ucdavisvillage.ddns.net:3334/</a>
4)	Obvius smart panel server	<a href="http://ucdavisvillage.ddns.net">http://ucdavisvillage.ddns.net</a>
5)	Battery data server via FTP	<a href="FTP://ucdavisvillage.ddns.net">FTP://ucdavisvillage.ddns.net</a>

## Database

For easier management of the data collected from the services, MongoDB and Heroku applications are used to structure the data into JavaScript Object Notation (JSON), snapshots are shown in Figure 5. A java based script transforms raw data from the data acquisition services to JSON. An example JSON is shown in Figure 5. MongoDB is used to manage the data from control mode 1 and 2; Heroku application is used for mode 3.



**Figure 5 (a) mongodb interface with stored data, (b) Heroku application**

## Energy Management

The battery pack was operated as an energy buffer shifting energy from times of peak PV production to times of peak energy consumption. The battery charge versus discharge decision was made based on three system variables: 1) battery status, 2) time varying utility price, and 3)



energy demand less the PV production. An example daily usage cycle typically has PV production occurring from 9am to 6pm, and any excess production will be stored in the battery pack. The typical energy usage peak occurs from 5pm to 9pm and typical utility time varying price peaks from 2pm to 8pm. During peak usage and peak utility price time periods, the battery tends to discharge to support the energy deficit. A detailed system energy flow management decision table is presented in **Error! Not a valid bookmark self-reference.**, where row 1, 2 and 3 are input variables and row 4 is a list of system actions. This energy flow management approach is a mild strategy in terms of utilizing battery storage. When utility price is off peak, the energy demand will always be covered by the grid instead of battery.

**Table 2 Decision table of controlling energy flow**

In1:	T	T	T	T	T	T	F	F	F	F	F	F
In2:	T	T	T	F	F	F	T	T	T	F	F	F
In3:	T	N	F	T	N	F	T	N	F	T	N	F
Md1:	I	I	C	D	D	I	C	C	C	I	I	I
Md2:	C	C	C	D	D	I	C	C	C	D	D	I
Md3:	I	C	C	D	D	I	C	C	C	D	I	I
Input In1 Utility Price: T, Peak Pricing; F, Off Peak In2 Usage: T, PV Generation>Demand; F: PV Generation < Demand In3: Battery SoC: T, 80%~100%; N, TarSoC~80; F, 0%~TarSoC												
Action C, Battery Charge; D, Battery Discharge; I, Battery Idle												

The battery SoC limits have a varying operation boundary conditions: target SoC. This target SoC value marks the level of charge the battery pack should maintain at the end of the day. It is calculated every day at the evening when PV energy production is finished using the following equation:

$$\text{SoC}_{\text{Target}} = 60\% - 0.5 * \Delta\text{SoC}_{\text{Batt}} \quad (1)$$

where  $\text{SoC}_{\text{Target}}$  is the target SoC at the end of the day (23:00), and  $\Delta\text{SoC}_{\text{Batt}}$  is the variation of the battery SoC during the day (from dawn to 18:00). During the utility peak time, the battery pack will discharge to support the house load, but will not exceed the target SoC level, so that it will keep a good level of charge and also leave enough capacity for receiving excess energy production from the PV on the following day.

Three different modes of operation were studied in this project. The objective of the energy management Mode 1 is to optimize the amount of solar energy back feeding to the grid during the peak pricing hours while the Mode 2 is to support the house demand with minimum grid support possible. The objective of the energy management Mode 3 is to optimize the utilization of solar energy to support the house energy demand.

#### **4. PROJECT OUTCOMES**

##### **Task1-1: Optimizing Energy Management Strategies for Different Objectives Including Demand Response**

The research team included utility pricing information obtained from CAISO OASIS API using a custom build http request sub routine coded in JAVA, the detail of the API can be found in [http://www.caiso.com/Documents/InterfaceSpecifications-OASISv4\\_1\\_1.pdf](http://www.caiso.com/Documents/InterfaceSpecifications-OASISv4_1_1.pdf) and the energy management program developed by the team is available at <https://github.com/AntonioTong/aggiehome>. CASIO provides day-ahead market (DAM) pricing data in the units of \$/mWh. Although the DAM is not exposed to the residential customers yet it would be the energy pricing for the residential homes once they are equipped with the energy management system in the near future. Utilizing this pricing information, our system is able to perform simulated dynamic pricing demand response, by penalizing certain time of a day as the 'high demand', and send a portion of battery stored renewable energy back to the grid.

The objective of the energy management Mode 1 is to optimize the amount of solar energy back feeding to the grid during the peak pricing hours. Figure 6 shows four days of system demonstration using the Mode 1. The house energy demand showed an average of 500W flat rate (lighting, refrigerator, ventilation), and random usage peaks happened in daytime and early evening (entertainment, cooking, AC). The PV array production peaked from 9am to 4pm. From morning to late noon, the battery pack would generally charge, storing excessive energy from PV panels. From afternoon to early evening (2pm to 8pm), the utility was at peak pricing period and the PV production would be sent back to the grid when available. When solar power ceased to exist, the battery pack picked up as the energy source to support the house demand. In this operation mode, the battery was primarily used for shifting peak hour usage. As indicated in the demonstration, the house's peak hour usage was mostly covered by PV production, and only a small fraction needs to be supported by the battery during early evening. As a result, battery was mildly cycled, with a daily usage of 2.7kWh given the total electricity usage of 15.4 kWh.

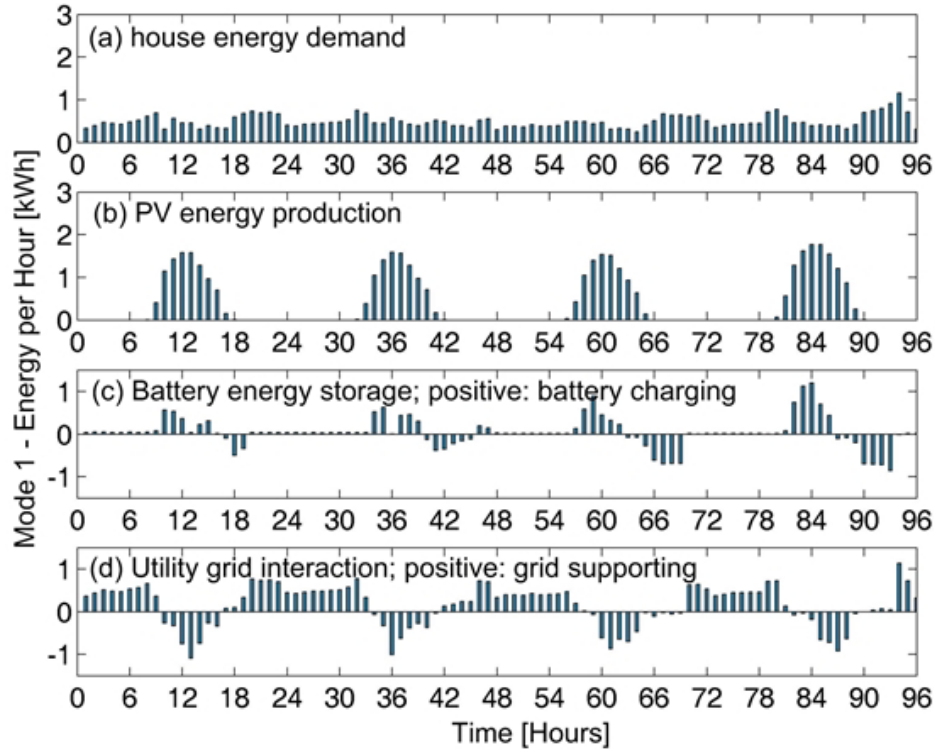
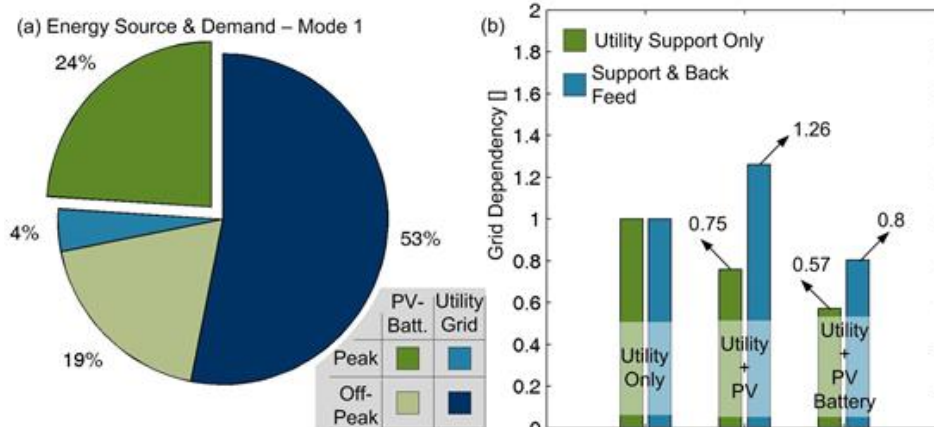


Figure 6. System operation with control Mode 1: (a) house energy demand, (b)solar panels energy production, (c) battery storage, (d) and utility interaction.

Figure 7(a) displays a pie-chart breaking down energy demands and sources. It indicates that about 28% of total usage was peak hour usage, and majority of it (86%) was covered by renewable sources. Figure 7(b) shows the grid dependency/usage of Mode 1. The green bar shows the grid usage level, which only considers utility support. And the blue bar shows the grid dependency level, which considers both support and back-feed as forms of grid interaction. Assuming that a house 100% powered by the utility has the grid dependency/usage of 1, as indicated by the bar charts, the Mode 1 reduced 43% of grid usage, and improved grid dependency by 20%. The Mode1 is a viable energy management method considering the typical peak pricing schedule. However, with improving solar penetration of the grid, sending back PV energy directly to the grid during the daytime will become less favorable.



**Figure 7. System operation summary with control Mode 1: (a) Energy source and demand pie-chart; (b) Utility dependency (green bar: utility support only; blue bar: utility interaction including support and back feed)**

The objective of the energy management Mode 2 is to support the house demand with the minimum grid support possible. Battery received PV energy during daytime at off-peak hour, and discharged whenever the house is in need. As shown in Fig. 8, during four days of 'low demand', the battery was able to cover all of the electricity usage in the house using Mode 2. As shown in Fig.9, both peak and off-peak usage were 100% supported by renewable energy, diminishing grid usage to 0. Note that over the four days of operation, the battery *SoC* was gradually depleting in order to support the demand of the house. The charge and discharge was not balanced. Once hit the lower *SoC* limit, the battery will be charged up with solar energy. The Mode 2 demonstration shows that the system can potentially support the entire house energy demand when the electricity demand is mild. If the system throttles the PV output instead of feeding back, Mode 2 will equip the house with an 'off-grid' capability, which minimizes the grid impact when needed.

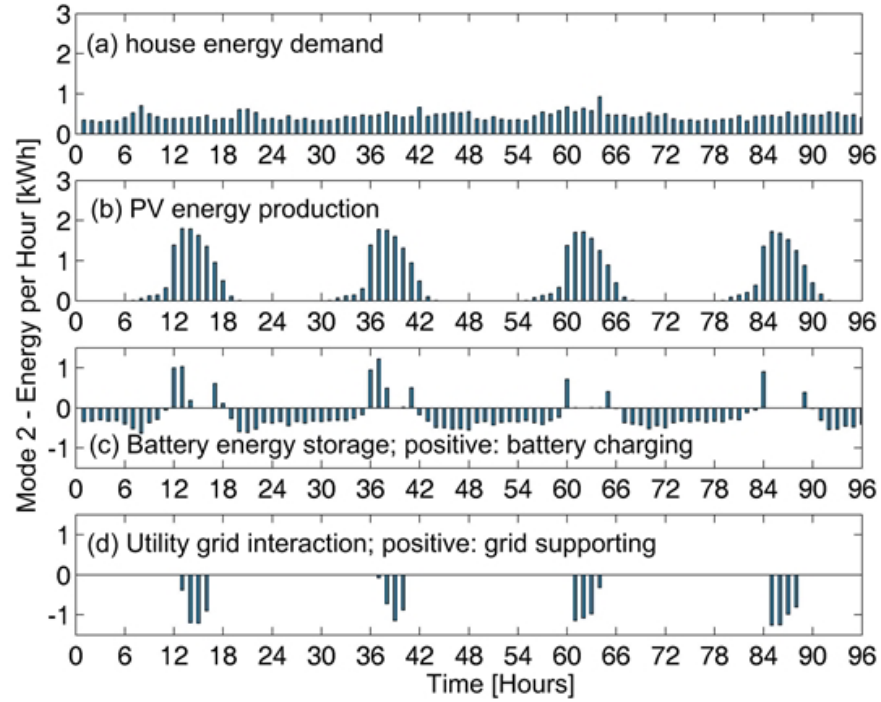


Figure 8. System operation with control Mode 2: (a) house energy demand, (b)solar panels energy production, (c) battery storage, (d) and utility interaction.

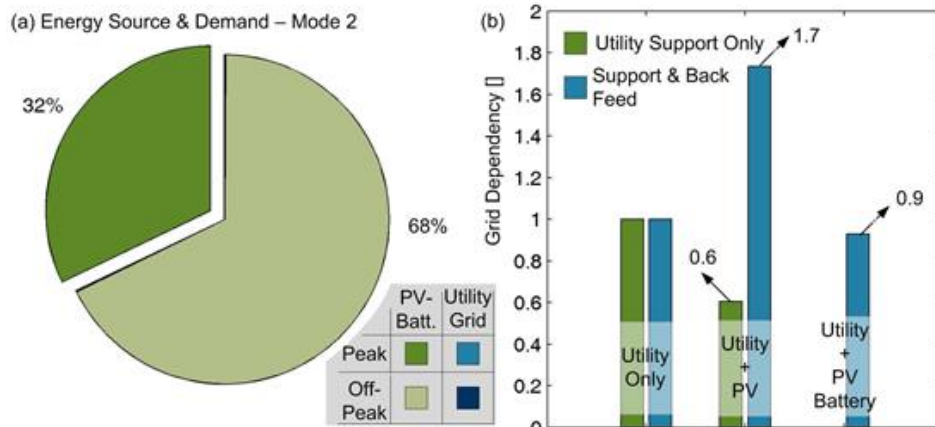
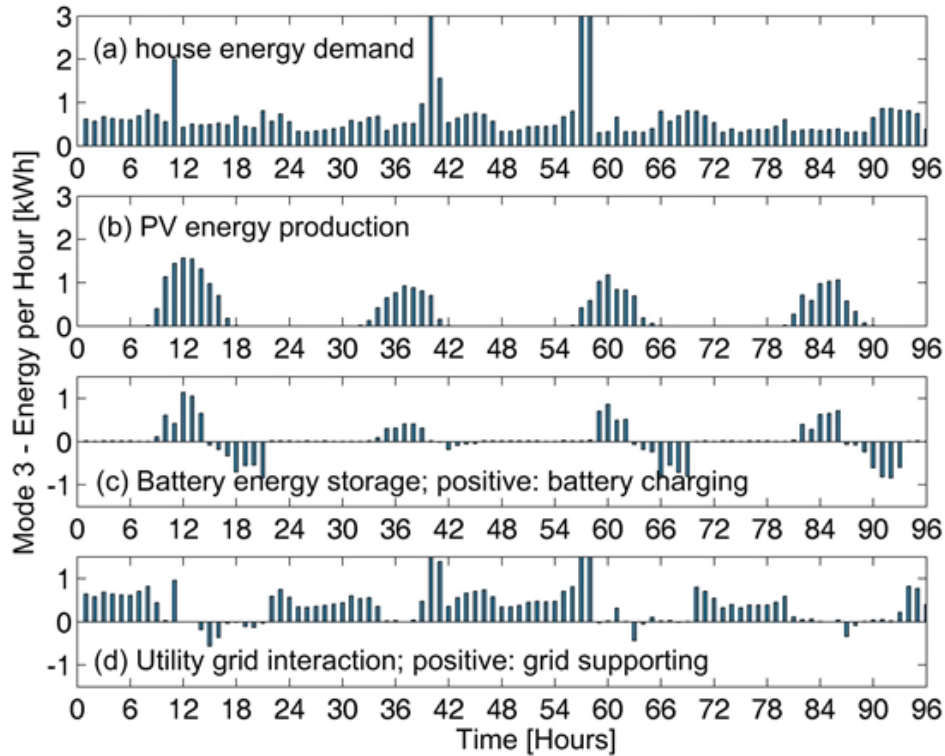


Figure 9. System operation summary with control Mode 2: (a) Energy source and demand pie-chart; (b) Utility dependency (green bar: utility support only; blue bar: utility interaction including support and back feed)

The objective of the energy management Mode 3 is to optimize the utilization of solar energy to support the house energy demand. As shown in Fig. 10, the battery pack was charged during the daytime both off-peak and peak hours, to store most of the PV over-production. During the nighttime, battery was discharged to support the peak usage. As a result, both the grid consumption and the back-feeding were reduced, but the solar penetration was enhanced. As

shown in Fig.11, Mode 3 covered 65% of the peak usage via renewable, reduced grid usage by 34% and improved grid dependency by 58%. Note that the peak usage coverage was not as high as Mode 1, that is because the energy demand during the Mode 3 demonstration was much higher, with several usage spikes that exceeded the battery discharge limit and had to be supported by the utility grid. Overall, the Mode 3 aggressively stored solar energy to neutralize the grid interaction instead of directly sending back to the grid. It will be a relatively favorable energy management approach for the high solar penetration future grid.



**Figure 10. System operation with control Mode 3: (a) house energy demand, (b)solar panels energy production, (c) battery storage, (d) and utility interaction**

This study demonstrated that 2<sup>nd</sup> life batteries can be re-purposed as stationary storage for solar energy storage and demand side management. But the commercial viability of this application is dependent on battery price, the grid storage market and the system long term performance to justify. Longer time period battery life cycle assessment testing will be conducted as an extension to this study. The size of the interconnected system was very small. With only one single family home and small scale roof top PV, the energy source and demand were constantly fluctuating subjecting to various impacts. It makes exploring the potential of energy storage vary challenging. Based on this study, three recommendation can be made: 1) 2<sup>nd</sup> life batteries can be re-proposed

as stationary energy storage given enhanced battery management to resolve issues in performance imbalance; 2) using battery module as the smallest building block, and deploying, cycling, and retiring the module independently, rather than replace the entire pack, will help reduce the system cost while maintaining the overall system performance; 3) larger size (community to micro-grid scale) distributed energy source and demand are preferred to optimize the performance of a 2<sup>nd</sup> life battery pack.

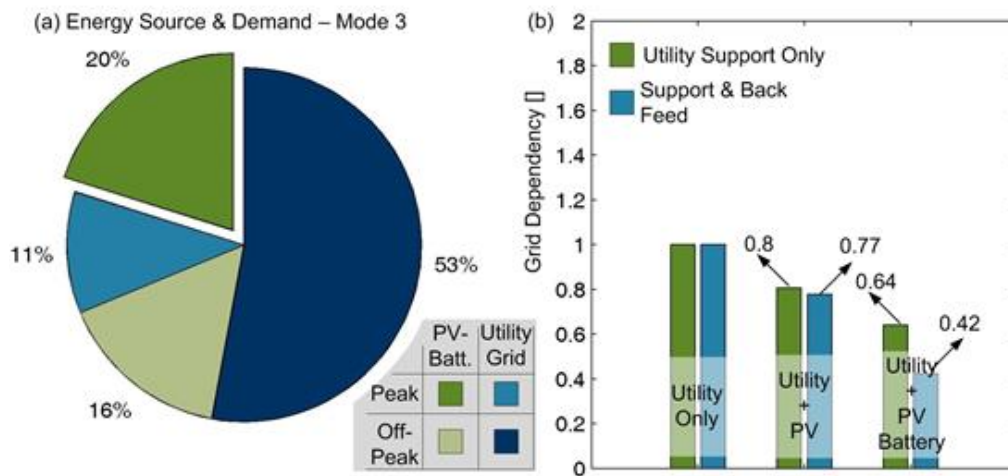


Figure 11. System operation summary with control Mode 3: (a) Energy source and demand pie-chart; (b) Utility dependency (green bar: utility support only; blue bar: utility interaction including support and back feed)

### Task1-2: Demonstrate the System with Modes 1, 2, and 3 for Long-Term Periods

The system has been fully functioning with data logger recording usage history. We have obtained number of series of long term operation data for different seasons. Figure 12 shows 6 weeks of system operation data in winter season. Due to the cloudy winter weather, the PV system is outputting energy about 4 to 7 kWh per day on sunny days. Figure 8 shows PV production summary from 1/19/2015 to 3/2/2015. Note that certain days the energy output is less than 1 kWh, that is due to some communication malfunction on the TiGo Solar Maximizer, resulting in one of the PV panel have no output occasionally, this issue was resolved before January 27th 2015. Aside of that, the PV array is working properly.



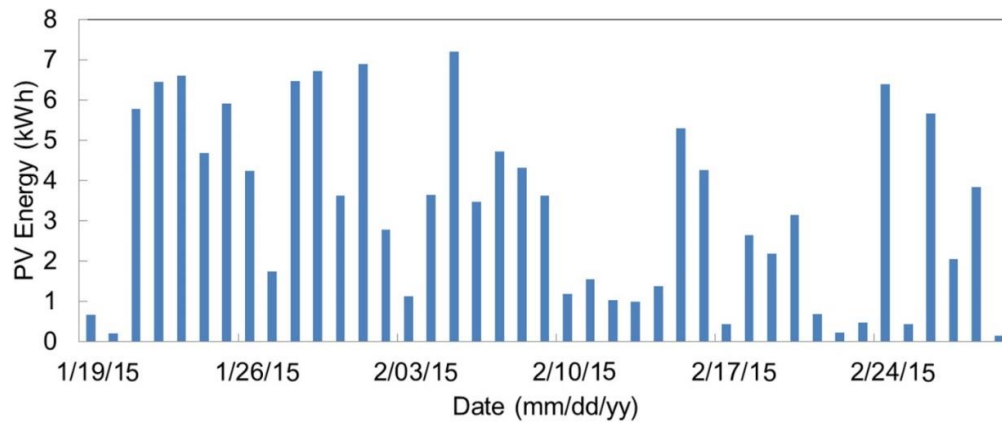


Figure 12. Example long term operation data PV Array Energy Harvesting Summary (1/19/2015 to 3/2/2015)

PV energy harvesting data in summer is consistent due to good weather. The system has been under long term operation with more data being collected, especially during sunny days when PV production is much higher than winter time. Fundamental data validates that the overall system is fully functioning and battery is controlled according to the energy management algorithm 1, 2 and 3.

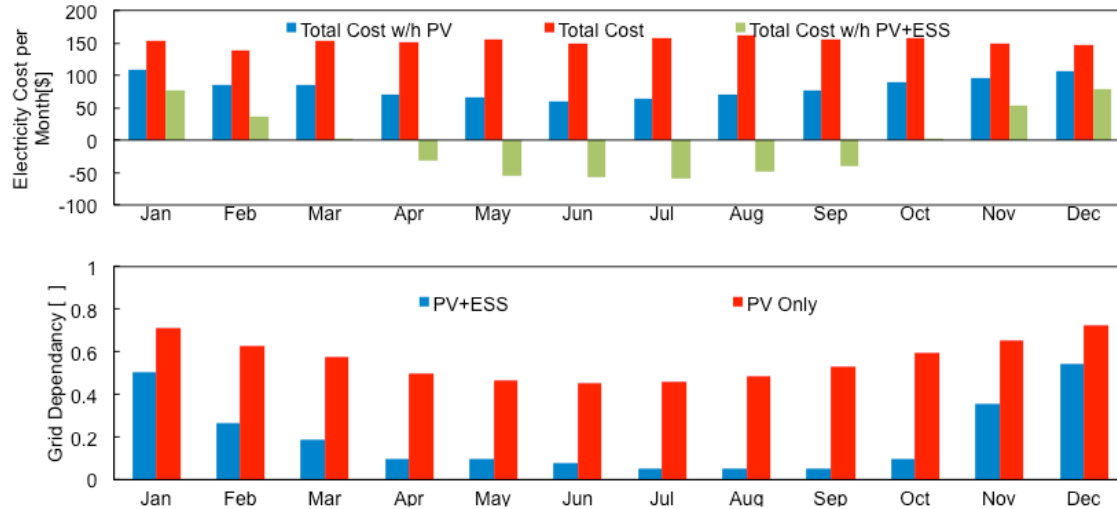
#### Task 2-1: Assessment of the System Cost, Benefit, and Environment Impact

Software has been developed that can evaluate the system cost, benefit and environment impact focusing on reduction of greenhouse gas emissions. The economic assessment was based on monthly electricity bills, PV energy production and EES system operation. Table 3 presents energy use, billing data, and estimated energy yearly bill according to different size of EES system. Our system corresponds to the result of 8 kWh.

Table 3 System assessment (economic)

Kilowatts Used in Each Billing Period for 2014				
Capacity of the Battery (kWh)	0	2	5	8
Summer Peak	626	380	187	93
Summer Partial-Peak	473	378	250	202
Summer Off-Peak	1,227	1,593	1,929	2,077
Winter Partial-Peak	339	110	18	2
Winter Off-Peak	1,656	1,904	2,002	2,021
Total Energy Use	4,322	4,365	4,386	4,395
Estimated Yearly Bill	\$556	\$508	\$466	\$447





**Figure 13 Monthly electricity cost estimation with 2.16 kW PV and 10kWh ESS**

Figure 13 shows the comparison of monthly electricity cost estimations between grid support only (red), grid support with PV generation (blue) and grid support with PV generation and EES energy management (green). Electricity cost from the grid is estimated as \$0.0705 per kWh according to the UC Davis utility board. During daytime, a portion of the demand is neutralized by PV production and the excessive production will be sent back to the grid @ \$0.03 per kWh, which is the typical solar payback rating. With the support of ESS and proper energy management control, the Aggie Village home is run by maximum renewable generation up to 100 % as already shown in Fig 6. The EES and energy management system not only maximizes the use of renewable generation but also can sell the excessive production (or even some portion of PV generation) through the DR or CAISO market at a higher rebate rate. We assumed that best rebate rate of \$0.0705 per kWh is available in this simulation. As a result, the home could maximize the daily operating value minimizing grid dependency at the same time. Especially during summer, the home could generate noticeable economic profits utilizing over-generation. It should be noted that, without the support of EES and grid control, RMI microgrid is still dependent on grid electricity even with significant over generation. In addition, CPUC proceeding. ([docs.cpuc.ca.gov/PublishedDocs/Published/G000/M159/K984/159984285.docx](https://docs.cpuc.ca.gov/PublishedDocs/Published/G000/M159/K984/159984285.docx)) is outing a new estimation methodology for determining Net Energy Metering (NEM) billing credits. Although our work was done before this, but our analysis is in line with CPUC's interest. For all of the test modes simulated, the energy storage is primarily used for energy time shifting of solar, and feed back to the exporting to the grid are essentially 'excess solar power'. As a result, this study should

fit into the estimation methodology that are under discussion at CPUC, and eligible for NEM credits without hitting the cap.

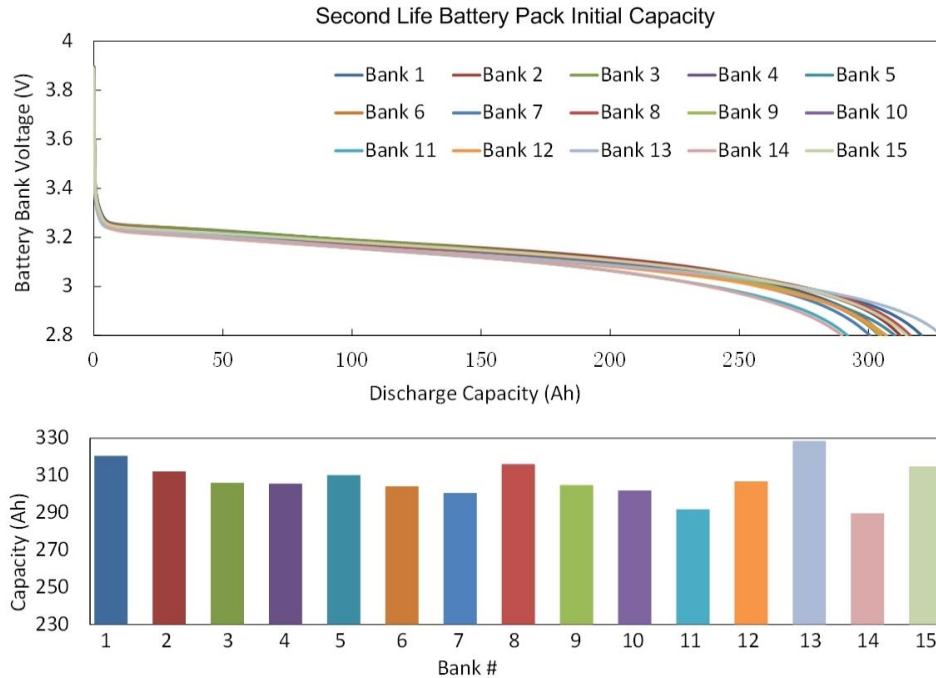
Table 4 summarizes the economic and environmental benefits of the developed EES system. Annual electricity cost saving is over \$3669.21 when 10 kWh EES is installed. This is equivalent to 100% reduction in electricity bill when compared to without PV support. The PV – EES system also brings a substantial reduction in CO2 emissions, up to 375 tons per year.

**Table 4. Summary of economic and environmental benefits of aggie home nano-grid**

Use from Grid [kWh]*	Electricity Use from PV [kWh]	Use from Battery [kWh]	Electricity Cost [\$]	Solar+ Battery Cost [\$]	Total Cost [\$]	Energy Saving perYear [\$]	CO2 Saving perYear [ton]**
<b>Yearly Electricity Cost</b>							
3651.12			1825		1825		
<b>Yearly Electricity Cost w/h 2.5 kW PV</b>							
2053.79	1596.21		1026.89	-50.9	976.00	1596.21	1.10
<b>Yearly Electricity Cost w/h 2.5 kW PV + 10 kWh Battery</b>							
19.21	1596.21	2073.00	-9.6	-35.15	-44.76	3669.21	2.53
*Utility cost is \$0.5/kWh							
** Emission rate $6.89551 \times 10^{-4}$ metric tons CO <sub>2</sub> / kWh ( <a href="http://www.epa.gov/cleanenergy/energy-resources/refs.html">http://www.epa.gov/cleanenergy/energy-resources/refs.html</a> )							

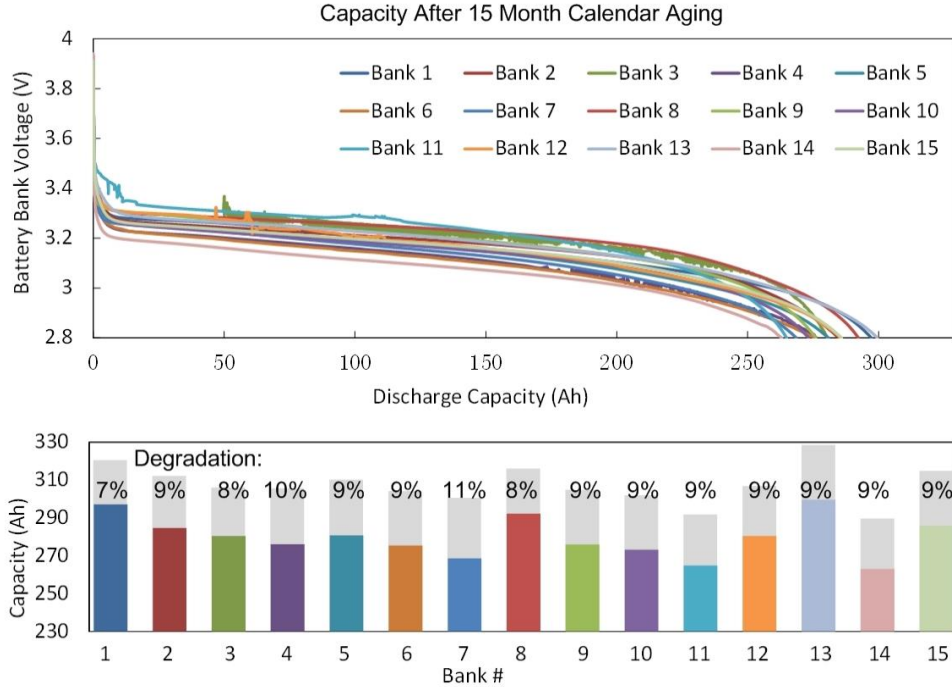
## **Task2-2: Assessment of the Second Life Battery Pack State of Health Degradation**

Among all the reconfiguration effort to design a second life battery pack, the most important task was custom design of an estimator that is able to accurately estimate both the SoC and SoH of all 15 battery banks while the system is under operation. SoC and SoH of the battery pack, which during dynamic operation may not be directly measured, are important battery state variables that are needed for battery management. For this battery pack, a multiple-time-scales worst-difference estimation approach was applied for SoC and SoH estimation.



**Figure 14. Initial capacity test results of 15 battery banks, each bank has nine cells connected in parallel. Top: Battery banks terminal voltage plotted against discharge; Bottom: Battery banks capacity in Ah**

Prior to assembling the battery pack, 15 battery banks were individually tested to quantify their capacity. As shown in Fig. 14 the 15 banks possess different useable capacities, the best battery bank being #13, which has a useable capacity of 328Ah, while the worst battery bank is #14, providing 287Ah.



**Figure 15. Capacity test of 15 month aging: Battery banks capacity in Ah**

The on-line state of health estimation results are presented in Figure 15. Applying the proposed battery management system, we are able to identify SoC and SoH of each individual battery bank after about 15 months of system operation. The battery estimation algorithm was able to estimate SoC of all 15 banks and successfully identify their differences during cycling. **Error! Reference source not found.**15 shows that the largest SoH degradation among banks was about 11% of the bank #7 while the minimum degradation was found in bank #1 as 7 %.

## 5. CONCLUSIONS

This project presents the development and the demonstration of a PV battery integrated energy system performing solar storage and demand side management in a single family home. The system provided a proof of concept that retired EV batteries can be reused for secondary applications, benefiting both transportation and building energy sectors. Three decision-table-based control strategies were demonstrated to test system functionality with different objectives. All three management modes achieved a significantly reduced grid dependency and improved solar penetration. Mode #1 has the modest battery daily usage which helps prolong the battery service life. Mode #2 has the most aggressive battery daily usage resulting in the minimum grid dependency. It also provides the most environmental and economic benefits. Mode #3 has the medium battery daily usage, but store solar energy to neutralize the grid interaction instead of

directly sending back to the grid. It will be a relatively favorable energy management approach for the high solar penetration future grid.

With proper design and engineering, used lithium batteries can be re-utilized as battery assemblies of competitive performance with the exception of imbalance at high state of charge, and lower round trip efficiency compared to systems with new batteries. A complete design document was presented including battery pack design and system integration. A battery management system was developed specifically for the 2nd life battery pack. It utilized enhanced balancing circuitries to improve the battery pack performance.

## **6. RECOMMENDATIONS**

This project demonstrated that 2nd life batteries can be repurposed as stationary storage for solar energy storage and demand side management. But the commercial viability of this application is up to the battery price, the grid storage market and the system long term performance to justify. Longer time period battery life cycle assessment concerned testing shall be conducted as an extension to this study. The size of the interconnected system was very small, with only one single family home and small scale roof top PV, the energy source and demand were constantly fluctuating in response to various impacts, making the exploration of the potential of energy storage very challenging. Based on this study, three recommendation can be made: 1) 2nd life batteries may be repurposed as stationary energy storage given enhanced battery management to resolve issues in performance imbalance; 2) using battery module as the smallest building block, and deploying, cycling, and retiring the module independently help reduce the system cost while maintaining the overall system performance; 3) larger size (community to micro-grid scale) distributed energy source and demand are preferred to optimize the performance of a 2nd life battery pack.

## **7. PUBLIC BENEFITS TO CALIFORNIA**

Benefits of the developed system include energy savings (economic), reduced CO<sub>2</sub> emissions (environmental), and enhanced grid stability and efficiency (technical). The system was able to achieve 64% to 100% reduction of grid usage, and further improvement of solar penetration, with the help of battery storage matching the source and the demand. As summarized in Table 4, annual electricity cost saving is over \$ 3,700 and annual reduction in CO<sub>2</sub> emission is up to 2.53 tons per year. The system also brought significant enhancement in grid stability and efficiency enabling demand response.

California has set an ambitious goal of having 33% of its electricity generation to be provided by renewable sources by 2020. However, due to the instabilities of wind and solar, energy storage will be an important component to enabling the meeting of this target. Energy storage enables the stable use of renewables for peak shaving, which can dramatically reduce the overall pollution caused by electricity generation as this is the critical period in which “peaker” plants, which generate the highest level of emissions, are operated. Finally, applying the use of second life lithium ion batteries for renewable storage has great potential when applied as distributed energy storage solutions at the site of renewable generation, for example when applied to residential homes as performed on this project.

The developed system will also have an impact on the commercialization of electric vehicle (EV) and plug-in hybrid electric vehicle (PHEV) technology. Although EVs and PHEVs are widely accepted as a promising technology for the near future, the advancement of a fleet of EVs and PHEVs still brings about significant challenges on GHG emissions since current methods of electricity generation rely heavily on carbon-intensive fuel sources such as petroleum, coal and natural gas. The use of electric power in transportation has a significant impact on GHG emissions in California if alternative energy sources are explored. Another challenge facing EVs and PHEVs is the high cost of the traction battery. The high cost of traction batteries has influenced automakers to pursue ideas for “second-life” applications of those batteries. A second-life battery could conceivably be used for non-traction applications when diminished energy storage capacity has rendered the battery unsuitable for providing power directly to a vehicle.

## References

1. Agency, International Energy. "Trends in Photovoltaic Applications: Survey Report of Selected Iea Countries between 1992 and 2009.". (2010).
2. Bragard, M., N. Soltan, S. Thomas, and R. W. De Doncker. "The Balance of Renewable Sources and User Demands in Grids: Power Electronics for Modular Battery Energy Storage Systems." *Power Electronics, IEEE Transactions on* 25, no. 12 (2010): 3049-56.
3. Doughty, Daniel H, Paul C Butler, Abbas A Akhil, Nancy H Clark, and John D Boyes. "Batteries for Large-Scale Stationary Electrical Energy Storage." *The Electrochemical Society Interface* (2010): 49-53.
4. Duryea, S., S. Islam, and W. Lawrance. "A Battery Management System for Stand-Alone Photovoltaic Energy Systems." *Industry Applications Magazine, IEEE* 7, no. 3 (2001): 67-72.
5. Ekren, O., and B. Y. Ekren. "Size Optimization of a Pv/Wind Hybrid Energy Conversion System with Battery Storage Using Simulated Annealing." [In English]. *Applied Energy* 87, no. 2 (Feb 10 2010): 592-98.

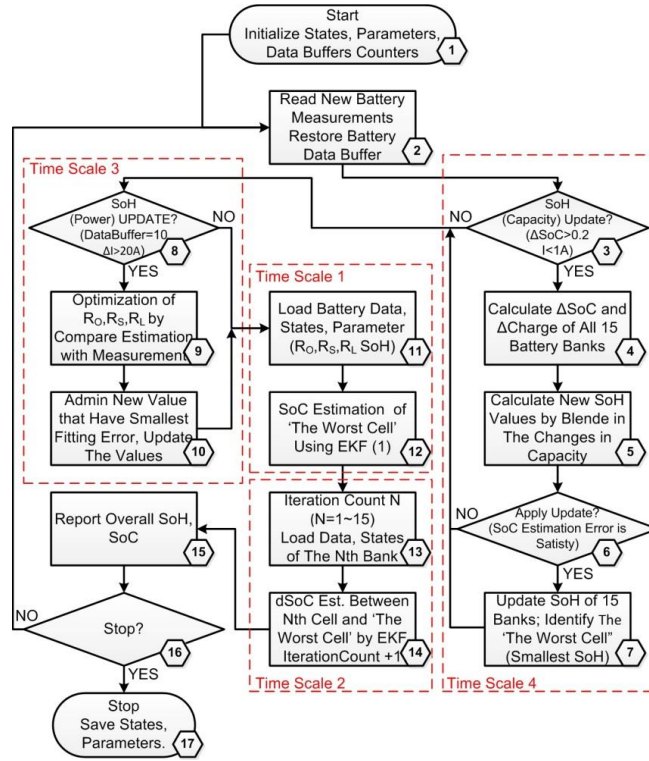
6. Ferreira, Summer , Wes Baca, Tom Hund, and David Rose. "Life Cycle Testing and Evaluation of Energy Storage Devices." ESS Peer Review Sandia National Laboratories (2012).
7. Haihua, Zhou, T. Bhattacharya, Tran Duong, T. S. T. Siew, and A. M. Khambadkone. "Composite Energy Storage System Involving Battery and Ultracapacitor with Dynamic Energy Management in Microgrid Applications." Power Electronics, IEEE Transactions on 26, no. 3 (2011): 923-30.
8. Hill, C. A., M. C. Such, Chen Dongmei, J. Gonzalez, and W. M. Grady. "Battery Energy Storage for Enabling Integration of Distributed Solar Power Generation." Smart Grid, IEEE Transactions on 3, no. 2 (2012): 850-57.
9. Hund, Tom , Nancy Clark, and Wes Baca. "Testing and Evaluation of Energy Storage Devices." ESS Peer Review Sandia National Laboratories (2008).
10. Jacobson, M. Z. "Review of Solutions to Global Warming, Air Pollution, and Energy Security." Energy & Environmental Science 2, no. 2 (2009): 148-73.
11. Landgrebe, Albert R., and Samuel W. Donley. "Battery Storage in Residential Applications of Energy from Photovoltaic Sources." Applied Energy 15, no. 2 (/ / 1983): 127-37.
12. Neubauer, J., and A. Pesaran. "The Ability of Battery Second Use Strategies to Impact Plug-in Electric Vehicle Prices and Serve Utility Energy Storage Applications." Journal of Power Sources 196, no. 23 (Dec 1 2011): 10351-58.
13. Omran, W. A., M. Kazerani, and M. M. A. Salama. "Investigation of Methods for Reduction of Power Fluctuations Generated from Large Grid-Connected Photovoltaic Systems." Energy Conversion, IEEE Transactions on 26, no. 1 (2011): 318-27.
14. Ongaro, F., S. Saggini, and P. Mattavelli. "Li-Ion Battery-Supercapacitor Hybrid Storage System for a Long Lifetime, Photovoltaic-Based Wireless Sensor Network." Power Electronics, IEEE Transactions on 27, no. 9 (2012): 3944-52.
15. Rydh, C. J., and B. A. Sanden. "Energy Analysis of Batteries in Photovoltaic Systems. Part II: Energy Return Factors and Overall Battery Efficiencies." Energy Conversion and Management 46, no. 11-12 (Jul 2005): 1980-2000.
16. Samaras, Constantine, and Kyle Meisterling. "Life Cycle Assessment of Greenhouse Gas Emissions from Plug-in Hybrid Vehicles: Implications for Policy." Environmental Science & Technology 42, no. 9 (2008/05/01 2008): 3170-76.
17. Schoenung, Susan. "Energy Storage Systems Cost Update." SAND2011-2730 (2011).
18. Tong, Shi Jie, Adam Same, Mark A Kootstra, and Jae Wan Park. "Off-Grid Photovoltaic Vehicle Charge Using Second Life Lithium Batteries: An Experimental and Numerical Investigation." Applied Energy 104 (2013): 740-50.

## Appendix

### Appendix I. Summary of EKF for battery SoC and deltaSoC estimation

In general, the proposed SoC/SoH estimation scheme identifies the worst battery bank in the pack, which has the smallest capacity, and allocates the available computing resources to provide close monitoring SoC and SoH of the worst bank. As for the rest of the banks, the scheme estimates their SoC and SoH by comparing them to the worst bank, significantly reducing computing resource demands. Figure A1 summarizes the flow chart of the scheme, which includes all the steps that are executed for a complete estimation cycle. It starts with initializing the state values, parameters and data buffers to be used in the scheme (step 1). At the beginning of the computing iteration (step 2), a fresh set of battery measurements are taken. Based on the knowledge of the battery pack, one bank will then be identified to be the worst battery bank (i.e. the lowest capacity). The SoH estimator then optimizes the battery capacity value of all 15 banks (step 3 to 7), at a frequency of Time Scale 4, using a varying parameter optimization approach. The SoH estimator also optimizes the battery internal resistance value of the worst bank based on cached measurement data (step 8 to 10), and this is processed at Time Scale 3. An Extended Kalman filter (EKF) was then applied to estimate the SoC of the worst bank (step 11,12) at Time Scale 1. Then, another EKF was applied to estimate the SoC difference between the worst bank and the rest of the banks at Time Scale 2 (step 13, 14). This estimator executes the estimation of three of the 15 banks during a single iteration, requiring five iterations to finish the estimation of all 15 banks in the pack.





**Figure A1. Flow chart of multiple time scales used for battery state estimation algorithm.**

In Figure A2, four different computing time scales are compared under the same time line to illustrate how the estimation algorithms are carried out. As time goes on, the SoC of the worst bank is estimated in each time step (Time Scale 1). The SoCs of the rest of the banks are estimated with a larger time scale (Time Scale 2), which updates after every five iterations. The internal resistance value of the worst bank updates after every five iterations (Time Scale 3). Finally, the capacity of all 15 battery banks are updated after every 1000 iterations. Capacity degradation is a slow procedure and therefore uses the longest time steps to quantify its variation. Over all, the estimation tasks on different battery banks of different states are composed into one integrated scheme, where the computational iteration is matched to the dynamics of each phenomenon.

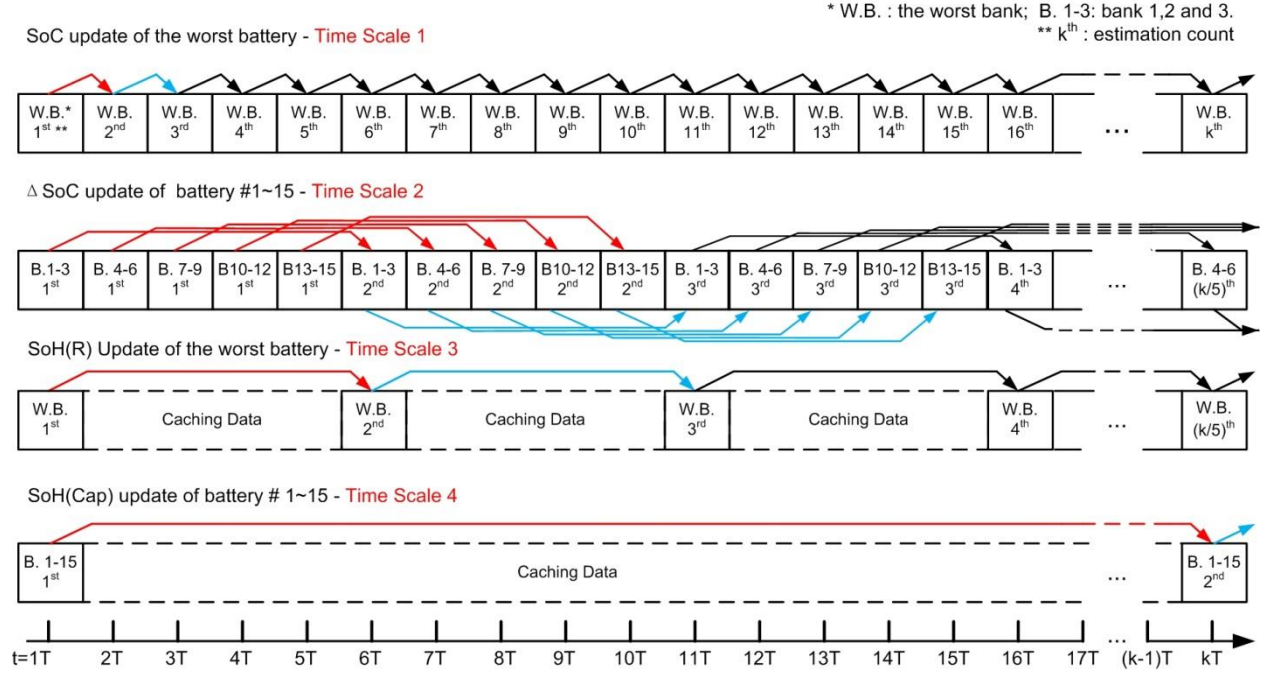


Figure A2. Diagram of multiple time scale battery state estimation algorithm.

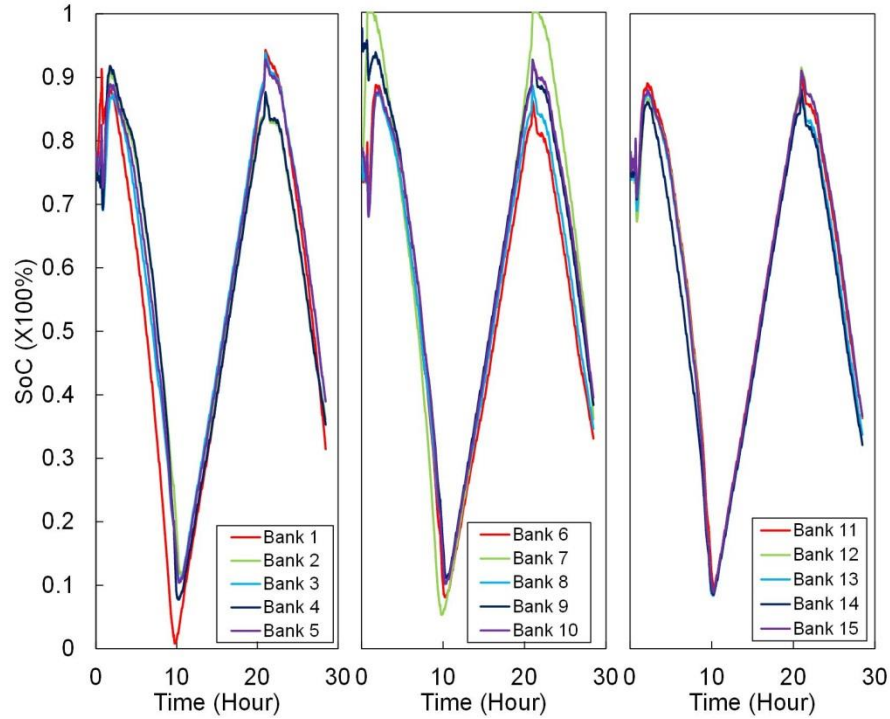


Figure A3. On-line battery SoC estimation during battery pack charge and discharge.

The battery pack has 15 banks in serial, each requires individual estimation of its SoC. A worst-difference estimation using EKF was developed for this task. Table A1 presents the state-space

model used for the worst battery bank. Table A2 presents the state-space model of the rest of the bank in comparison of the worst battery bank. Table A3 presents the computational steps used by the nonlinear extended Kalman filter.

**Table A1: State-space model used for the worst battery bank**

Computes each iteration  $\Delta t = 5s$

State and parameter variables

$$x = [SoC, U_s, U_L], \quad P = [R_s(1 - \exp(\frac{-\Delta t}{R_s C_s})), R_L(1 - \exp(\frac{-\Delta t}{R_L C_L})), R_{ohmic}]$$

Functions

$$f(x_k, u_k, P_k) = Ax + \begin{bmatrix} P(1) \\ P(2) \end{bmatrix} I$$

$$g(x_k, u_k, P_k) = OCV(SoC) + U_s + U_L + P(3)I$$

Where

$$A_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp(\frac{-\Delta t}{R_s C_s}) & 0 \\ 0 & 0 & \exp(\frac{-\Delta t}{R_L C_L}) \end{bmatrix}, \quad C_k = \begin{bmatrix} \frac{dOCV(SoC)}{SoC} \Big|_{SoC = SoC_k^-} & 1 & 1 \end{bmatrix}$$

$$C_k^P = [0 \ 0 \ 0 \ I_k]$$

$$+ \left[ \frac{dOCV}{dSoC} \ 1 \ 1 \right] \left\{ \begin{bmatrix} I_{k-1} & 0 & 0 & 0 \\ 0 & I_{k-1} & 0 & 0 \\ 0 & 0 & I_{k-1} & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp(\frac{-\Delta t}{R_s C_s}) & 0 \\ 0 & 0 & \exp(\frac{-\Delta t}{R_L C_L}) \end{bmatrix} \left( -L_{k-1}^x [0 \ 0 \ 0 \ I_{k-1}] \right) \right\}$$

Initialize with

$$\Sigma_w = [1; 0.0001; 0.001], \Sigma_v = 10, \Sigma_r = 0, \Sigma_e = 0$$

note that parameters other than P is not time varying;  $\tau_s = R_s C_s = 5, \tau_L = R_L C_L = 50$

**Table A2: Summary of state-state model for the worst battery bank**

For cell number  $k=1, \dots, 15$ , computes every 15 iteration  $\Delta t = 75s$

State and parameter variables

$$x = \Delta SoC, \quad P = \frac{\Delta t}{C_{NOM} SoH}$$

Functions

---


$$f(x_k, u_k, P_k) = Ax + \begin{bmatrix} P & -\frac{\Delta t}{C_{NOM} SoH_{ref}} \end{bmatrix} \begin{bmatrix} I \\ I_{ref} \end{bmatrix}$$

$$g(x_k, u_k, P_k) = OCV(x + SoC_{ref}) - OCV(SoC_{ref})$$

Where

$$A_k = 1, \quad C_k = \left. \frac{dOCV(SoC)}{dSoC} \right|_{SoC = SoC_k^-}, \quad C_k^P = C_k(I_k)$$

Initialize with

$$\Sigma_w = 1, \Sigma_v = 10, \Sigma_r = 0, \Sigma_e = 0$$


---

note that parameters other than P is not time varying

**Table A3: Summary of state space model for the difference between the worst banks and the rest of the banks**

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Nonlinear state-space model

$$x_{k+1} = f(x_k, u_k, P_k) + w_k$$

$$y_k = g(x_k, u_k, P_k) + v_k$$

Definition

$$A_{k-1} = \left. \frac{\partial f(x_{k-1}, u_{k-1}, P_k)}{\partial x_{k-1}} \right|_{x_{k-1} = \hat{x}_{K-1}^+}, C_k = \left. \frac{\partial g(x_k, u_k, P_k)}{\partial x_k} \right|_{x_k = \hat{x}_K^-}$$

Initialize with

$$\hat{x}_0^+ = E[x_0]$$

$$\Sigma_{x,0}^+ = E[(x_0 - \hat{x}_{K-1}^+)(x_0 - \hat{x}_{K-1}^+)^T]$$

Computation for  $k \in \{1, \dots, \infty\}$

State estimate time update:

$$\hat{x}_k^- = f(\hat{x}_{k-1}^+, u_k, \hat{P}_k^-)$$

$$\Sigma_{\hat{x},k}^- = A_{k-1} \Sigma_{\hat{x},k-1}^+ A_{k-1}^T + \Sigma_w$$

State estimate measurement update:

$$L_k^x = \Sigma_{\hat{x},k}^- (C_k^x)^T [C_k^x \Sigma_{\hat{x},k}^- (C_k^x)^T + \Sigma_v]^{-1}$$

$$\hat{x}_k^+ = \hat{x}_k^- + L_k^x [y_k - g(\hat{x}_k^-, u_k, \hat{P}_k^-)]$$

$$\Sigma_{\hat{x},k}^+ = (I - L_k^x C_k^x) \Sigma_{\hat{x},k}^-$$


---

where  $w, v$  are independent, zero-mean, Gaussian noise processes of covariance  $\Sigma_w, \Sigma_v$